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OPTIMAL RELEASE SEQUENCES FOR WATER QUALITY CONTROL IN MULTIPLE-RESERVOIR SYSTEMS

Norbert A. Jaworski, et al

Federal Water Pollution Control Administration
Annapolis, Maryland

September 1969

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Technical Report No. 11

OPTIMAL RELEASE SEQUENCES FOR WATER QUALITY CONTROL IN MULTIPLE-RESERVOIR SYSTEMS*

by

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September 1969

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INTRODUCTION

To better control, protect, and manage the water quality of our rivers, water resources engineers have directed increased attention to development of analytical techniques for the determination of procedures and policies by which optimal operation and management of entire river basins may be realized. Analytical approaches to the design and management of water resources systems have as their fundamental objective the provision of water at a preset quality for the least cost. These analyses also include investigation of alternatives, determination of inadequacy, probabilities, and establishment of optimal operational procedures.

There are essentially three methods for controlling water quality within a given river basin: (1) regulation of the total wastewater load entering the surface waters, (2) regulation of the river flow, either by reservoirs, pumping from well fields, or by pumped storage, in order to provide appropriate dilution of waste loads when required, and (3) utilization of in-stream treatment devices. The problem presented in this ~~is as follows~~ report concerns: given a basin with a developed or proposed impoundment system, develop a method for formulating reservoir operating policies for water quality control in an optimal manner.

Thomas and Watermayer [15] have summarized the research on that aspect of flow regulation relating to the probabilistic character of storage systems and dynamic decision making. Thomas describes the flow regulation problem as one of choosing: (1) the optimal operative policies, (2) the optimal level of development, and (3) the optimal reservoir capacities. The optimal conditions are defined in economic terms as the expected net benefits of the reservoir system.

Dorfman [8] has studied the relationships between design and operation decisions and deduced design from operating considerations. Thomas and Watermayer [15], utilizing linear programming, proposed a model for the reservoir operation problem. The model optimizes the operating policy and the levels of development, and by sampling, the reservoir capacity. Young [18] has developed two Monte Carlo techniques for finding the optimal annual operating policies for single reservoirs such that the economic loss as a function of draft is minimized.

Hall [5] has applied dynamic programming for allocating water for various purposes such as hydroelectric power generation, consumptive use, and water quality control. While the model proposed by Hall is limited to the economic considerations involved in the design of a reservoir, it does demonstrate the application of dynamic programming for designing multiple-purpose water projects.

Worley [17] has developed a general river basin model capable of determining and adjusting storage reservoir flows necessary to

maintain a minimum allowable dissolved oxygen concentration in a stream. The model is not an optimum-seeking model; that is, there is no mechanism incorporated to provide for the best operation of a reservoir system in terms of water quality management.

In 1966, a joint research project was initiated by the University of Michigan and Chesapeake Field Station*, Federal Water Pollution Control Administration [6], having the following objectives:

1. The development of a method for determining optimal flow regulation sequences for quality control from multiple reservoir sites,

2. The investigating of the various physical, biochemical, engineering design, and socio-economic parameters which may influence the optimal flow release sequence,

3. The investigating of the significance of various definitions of optimal reservoir release sequences, and

4. The demonstration of the response sensitivity of the method to these parameters in an actual basin.

An optimal release sequence for the initial phase of this study was defined as providing the "best" quality of water for a given flow rate at a point in the system while maintaining a minimum preset dissolved oxygen level in all reaches of the basin

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above this point. The "best" quality of water was further defined as the minimum DO deficit for a given concentration of BOD.

DYNAMIC PROGRAMMING SOLUTION

Various methods for solving the proposed problem were investigated, such as linear programming and dynamic programming. Due to the nature of the stream-flow network in a river basin, the use of dynamic programming to obtain an optimal solution was found to be an ideal approach.

The large complex release problem can be decomposed, called staging, into a series of smaller problems. This technique* relies upon decision-making at each stage rather than trying to solve the entire N-stage optimization problem simultaneously. A similar approach also has been suggested recently by Meier and Beightler [9].

There are two general approaches to the reservoir problem using dynamic programming:

1. starting at the mouth of the basin, proceeding upstream, and structuring the solution as a diverging branch system; or

2. starting at the uppermost point of the basin, proceeding downstream, and structuring the solution as a converging branch system. Since many of the water quality parameters are flow dependent, the solution was simplified by the latter approach.

An overall schematic diagram of a reservoir system such as proposed for the Potomac River Basin, shown in Figure 1, typifies converging structure. The reservoirs (triangles) are the controlling

* For a complete discussion of the dynamic programming technique, see Bellman and Dreyfus [1].

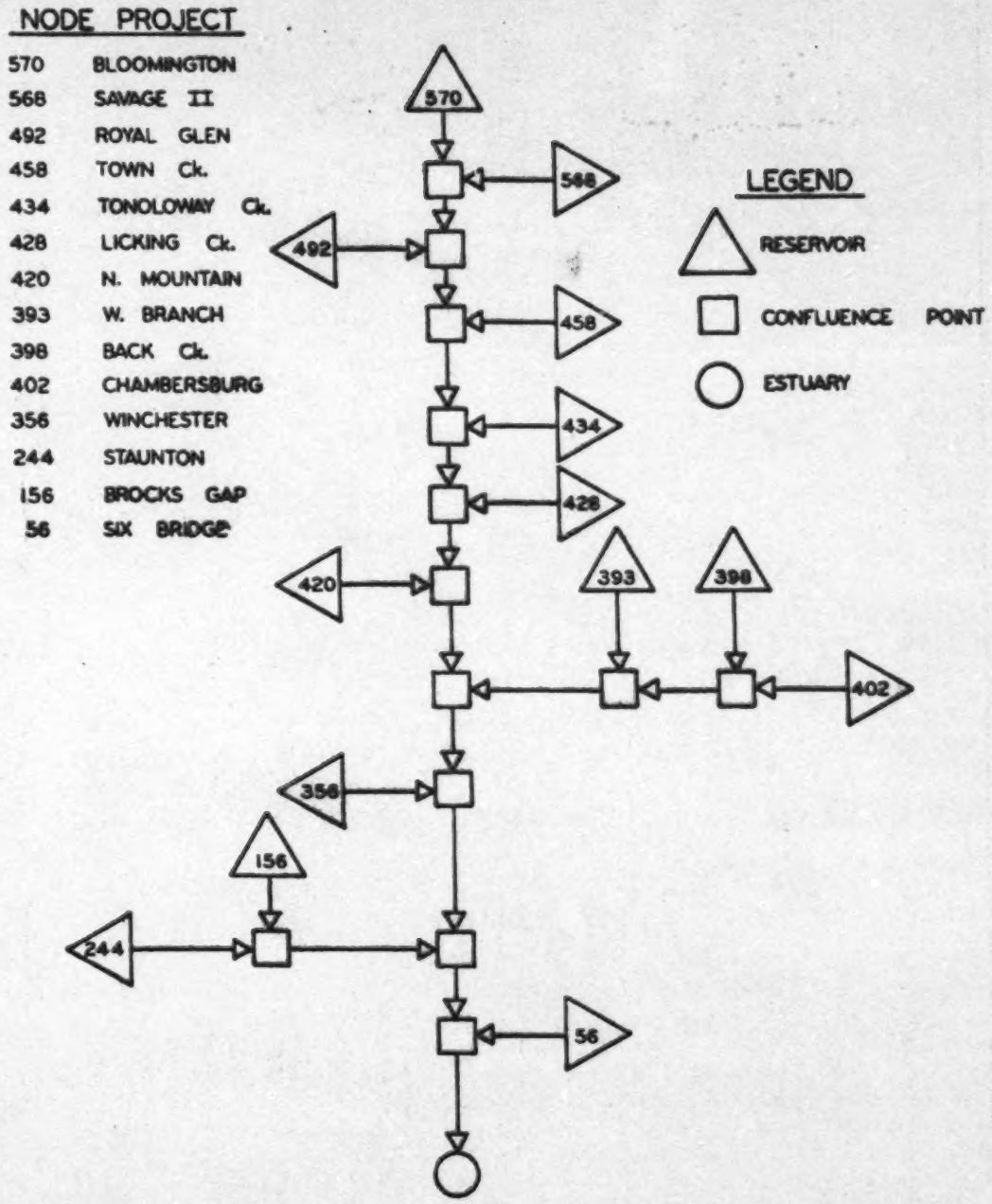


Figure 1.- A Converging-Branch System of Dynamic Programming Developed For the Potomac River Basin

mechanisms, and the regulated confluences (squares) are the decision points or the stages. In the Potomac system, the converging-branch structure results in a 13-multistage optimization problem. Presented in Figure 2 is a stage of a regulated river system characterized by six factors:

1. an input state, MFLOW, flow from a regulated system M, the dependent states being MBOD, MDOD, MK, and MT;

2. an input state, NFLOW, flow from a regulated system N, the dependent states being MBOD, MDOD, NK, and NT;

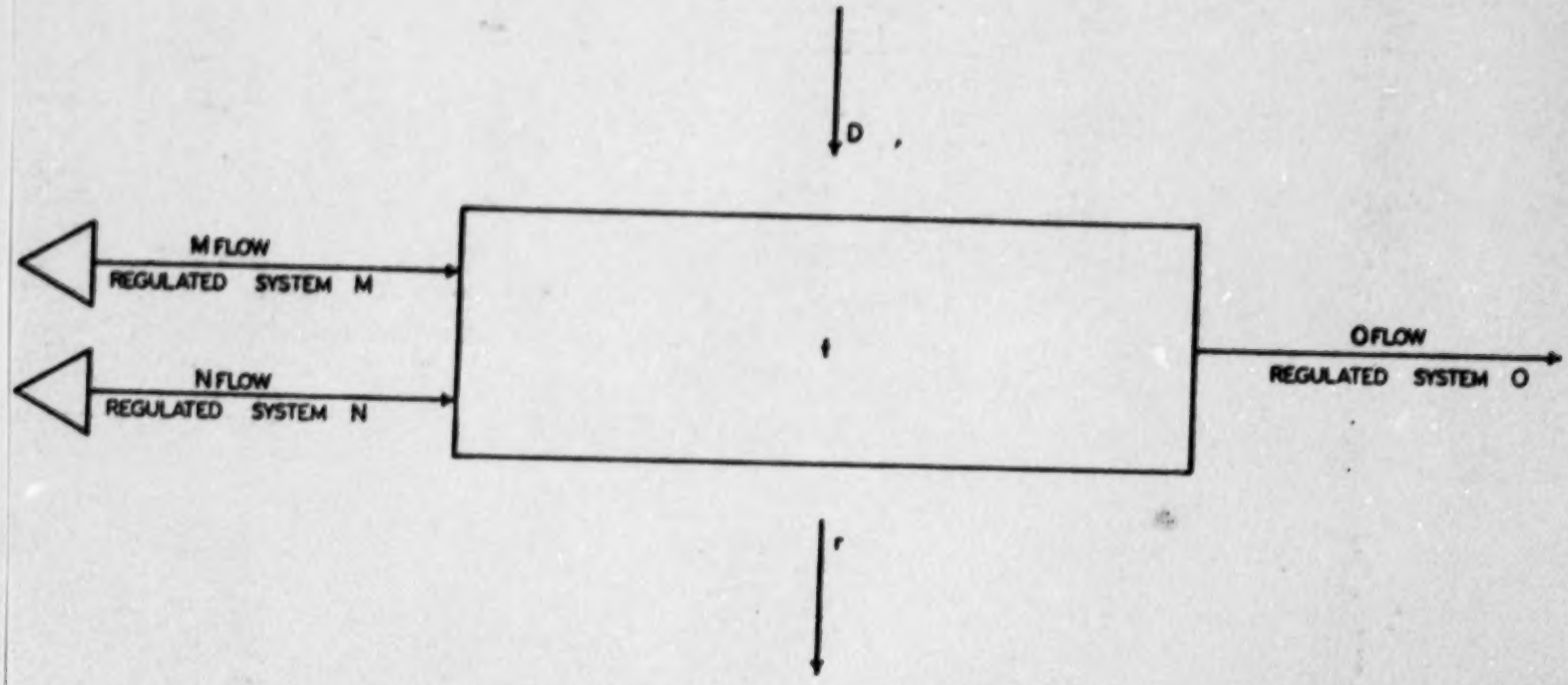
3. an output state, OFLOW, flow from the combined regulated systems of M and N, the values of the dependent states, OBOD, ODOD, OT, and OK, being a function of a combination of MFLOW, and NFLOW;

4. a decision variable D, dictates combination of NFLOW and MFLOW for a given state of OFLOW;

5. a stage return r, for a given OFLOW state, the best water quality measured in terms of minimum dissolved oxygen deficit ODOD for a given OBOD state; and

6. a stage transformation t, couples MFLOW and NFLOW and other dependent variables linearly.

The flow release problem is different from the general solution to multistage converging dynamic programming problems structured by Nemhauser [10] in three respects: (1) each stage consists of two input states, (2) flow release optimization is required for all output states in order to provide for regulation for quality control downstream if needed, and (3) the stage return is a dual-valued



M FLOW = INPUT FLOW STATE OF M

N FLOW = INPUT FLOW STATE OF N

O FLOW = OUTPUT FLOW STATE OF O

D = DECISION VARIABLE

r = STAGE RETURN

t = STAGE TRANSFORMATION

Figure 2 - A Stage of the Converging-Branch System

function, initially, the minimum ODOD for an OBOD state. The latter requires a decision procedure for two dependent states and a procedure for maintaining various feasible solutions per flow state.

Flow States

For a given stage or decision point, the range of OFLOW is dependent on the regulation capabilities of the two input systems M and N as shown in Figure 2. For a hypothetical example of a simple system (Figure 3), if the regulation capability of M is from 100 to 300 cfs and of N, from 200 to 500 cfs, the range of OFLOW would be from 300 (200 + 100) to 800 (300 + 500) cfs. To reduce the infinite number of input and output states, reservoir releases are made in discrete increments. In the hypothetical example as presented in Figure 3, if a 20 cfs increment is employed, there would be 26 states (800/20 - 300/20 + 1) of OFLOW.

The number of combinations of MFLOW and NFLOW which will equal a given OFLOW state is dependent on the value of OFLOW and the number of feasible solutions per states of MFLOW and NFLOW. The maximum number of combinations will be near the mean of the OFLOW range with a decreasing number towards the extremes of the range. As can be seen in Figure 3, for the simple two reservoir system, there are 11 combinations which equal 560 cfs while only two equal 320 cfs. For decision points in a more complex system in which there is more than one state of MFLOW or NFLOW, the number will increase proportionately.

N FLOW, FLOW STATE OF N (cfs) \ M FLOW, FLOW STATE OF M (cfs)	100	120	140	160	180	200	220	240	260	280	300
200	300	320	340	•	•	•	•	•	•	•	•
220	320	340	•	•	•	•	•	•	•	•	•
240	340	•	•	•	•	•	•	•	•	•	•
260	•	•	•	•	•	•	•	•	•	•	560
280	•	•	•	•	•	•	•	•	•	560	•
300	•	•	•	•	•	•	•	•	560	•	•
320	•	•	•	•	•	•	•	560	•	•	•
340	•	•	•	•	•	•	560	•	•	•	•
360	•	•	•	•	•	560	•	•	•	•	•
380	•	•	•	•	560	•	•	•	•	•	•
400	•	•	•	560	•	•	•	•	•	•	•
420	•	•	560	•	•	•	•	•	•	•	•
440	•	560	•	•	•	•	•	•	•	•	•
460	560	•	•	•	•	•	•	•	•	•	760
480	•	•	•	•	•	•	•	•	•	760	780
500	•	•	•	•	•	•	•	•	760	780	800

COMBINATIONS of MFLOW + NFLOW = (OFLOW = 560 cfs)

Figure 3 - Flow States For a Simple Hypothetical Converging-Branch System

Decision Process (Efficient Enumeration)

For the decision process of each stage, a technique was developed which would determine for a given OFLOW state and within a given OBOD grid, the combination of MFLOW and NFLOW that yields the minimum ODOD. To add greater versatility, an externally controlled grid, which was made a function of the OBOD range, was incorporated into the optimization algorithm. This permits the selection of one or more feasible solutions per OFLOW state. Combinations of MFLOW and NFLOW for a given OBOD grid increment with a greater ODOD are deleted.

For each state of OFLOW, starting with the minimum and progressing to the maximum regulation capability, the optimization algorithm, which is an efficient enumeration process, consists of the following steps:

1. calculate all dependent variables for all combinations of MFLOW and NFLOW which equal the given OFLOW state;

2. sort and rank upward all feasible solutions according to value of OBOD parameter;

3. determine OBOD grid size by subtracting minimum OBOD value from the maximum, comparing OBOD difference to various input grids, and selecting proper OBOD increment size;

4. for the flow combinations of MFLOW and NFLOW equaling the OFLOW state and the OBOD grid increment, select the combination which has the minimum ODOD;

5. increment to the next OBOD grid increment and repeat

Step 4 until all the optimal solutions are selected for all OBOD grid increments for the given flow state OFLOW; and

5. increment to next OFLOW state and repeat Steps 1 through 5 until all OFLOW states have been mapped.

An example of technique in which 11 combinations of NFLOW and MFLOW equaled a given OFLOW is given in Figure 4. The OBOD range was divided into three OBOD grid increments. For the OFLOW state, the number of feasible solutions was reduced from 11 to 3. As can readily be seen from the above six steps and Figures 3 and 4, this is a recursive process and very applicable to solution on a digital computer.

With the converging-branches approach, the flow regulation needs for the quality control reaches downstream from the decision point under investigation are unknown. Therefore, for a given stage, a range of feasible solutions for all states of OFLOW must be carried downstream.

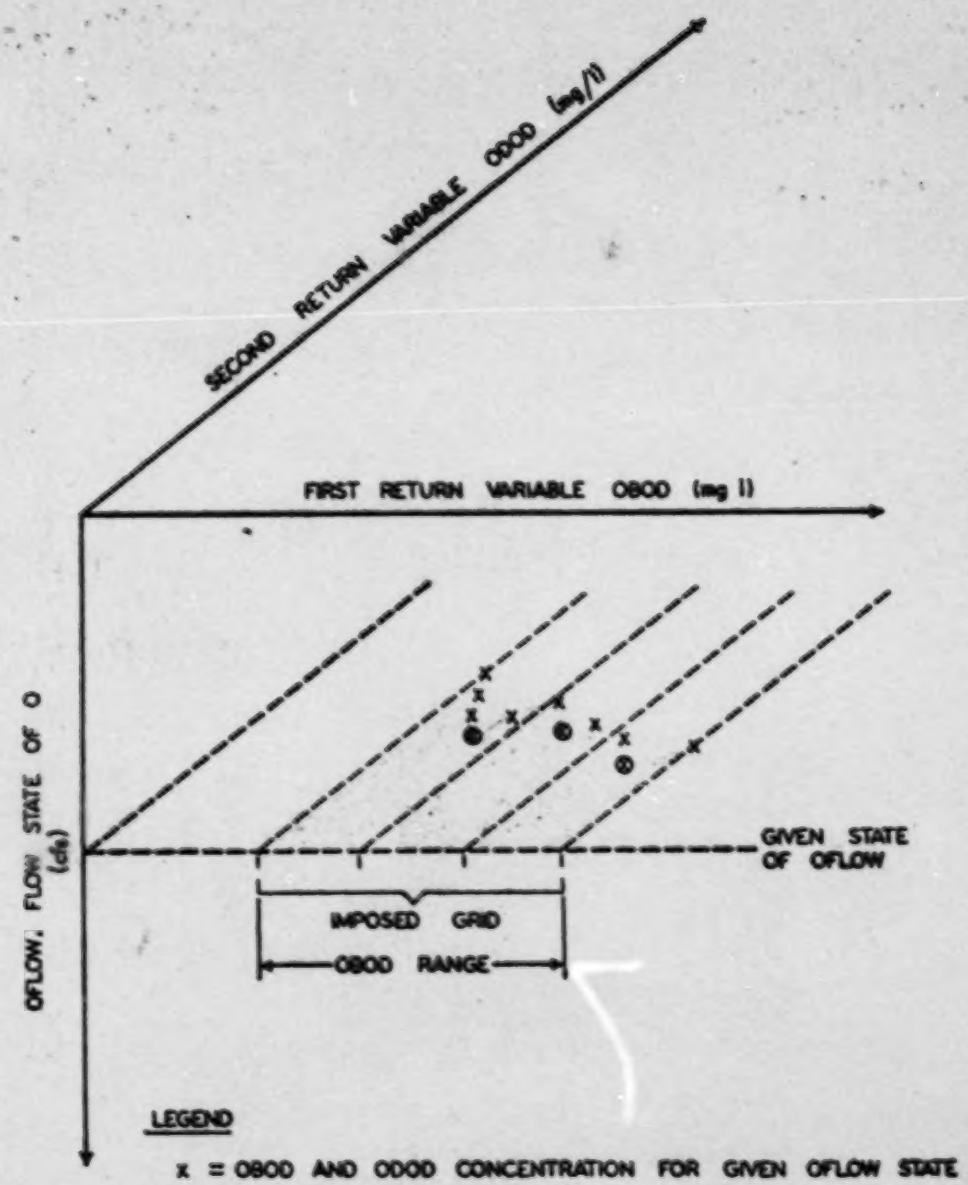

SECOND RETURN VARIABLE ODOD (mg/l)
FIRST RETURN VARIABLE OBOD (mg l)
OFLOW, FLOW STATE OF O (cfs)
GIVEN STATE OF OFLOW
IMPOSED GRID
OBOD RANGE
LEGEND
x = OBOD AND ODOD CONCENTRATION FOR GIVEN OFLOW STATE
⊛ = MINIMUM ODOD FOR GIVEN OBOD GRID INCREMENT

Figure 4 - An Example of the Decision Process Used in Selecting Optimal Solutions for a Given OFLOW State

Water Quality Formulations

Water quality formulations in the flow release model have been limited to temperature, DO, and BOD, with the main parameter being DO. The water quality algorithm used to predict the dissolved oxygen deficit, D (t), in the flowing stream, in the model is Thomas's modification [14] of the Streeter-Phelps equation as given below:

$$D(t) = Y * P\,D(0) * P^2 \qquad \text{- - - - - (1)}$$

Where Y = Oxygen Uptake Term

P = Reaeration Term

The reaeration rate, K_2, formulations employed are the non-isotrophic relationships developed by O'Connor [11]:

$$K_2 = \frac{(D_L V)1/2}{H^{3/2}} \qquad \text{- - - - - (2)}$$

and the Churchill formulation [3]:

$$K_2 = \frac{5.026_V 0.969}{H^{1.673}} \qquad \text{- - - - - (3)}$$

To maintain simplicity, the exponential decay temperature model of Duttweiler [4] has been incorporated into the overall algorithm. The temperature, T_1, at a point in the system, based on stream velocity, V, distance from heat source, X, and a temperature decay coefficient, W, is given below:

$$T_1 = Ts_1 + [Ts_0 - T_0]\, e^{-\frac{X}{WV}} \qquad \text{- - - - - (4)}$$

The corrections for the effects of temperature on the deaeration and reaeration rates, K_t, [12] employed in the water quality algorithms are given below:

$$K_t = K_{20} \times \theta^{(T-20)} \qquad \text{- - - - - (5)}$$

A third order polynomial for determining the solubility or dissolved oxygen in water at a given temperature, T, has been incorporated into the model. The polynomial coefficients used for determining DO at saturation temperatures, SAT DO, were those developed by Churchill [13], yielding:

$$\text{SAT DO} = 14.652 - 0.41022T + 0.00799107T^2 - 0.000077779T^3 \qquad \text{- - - - - (6)}$$

In the above dissolved oxygen, reaeration, and temperature formulations, physical parameters of the stream such as velocity, V, and depth, H, are required for various ranges of flow, Q. The characteristic relationships developed by Leopold and Maddox [7] for these physical stream geometry parameters were used as presented below:

$$H = aQ^b \qquad \text{- - - - - (7)}$$

$$V = cQ^d \qquad \text{- - - - - (8)}$$

Overall Flow-Release Algorithm and Computation Procedures

For the overall flow-release algorithm, a descriptive model was developed which mathematically links the water quality formulations, multistaging process and optimization procedure to the stream network of the river basin. The descriptive model also selects stream flow routing patterns, calculates the water quality conditions, checks for the DO constraints, and sequences the multistaging process for the dynamic programming technique.

The basic computational steps of the flow release model are given below:

1. read in and display input data;

2. determine minimum and maximum stream flow ranges for each node in the basin required in the stream flow routing and multistaging process;

3. route the unregulated stream flows including water quality conditions to regulated sections of the basin;

4. starting at the uppermost decision point, route the stream flow including water quality parameters by increments for entire range of regulation capacities of the two contributing reservoirs;

5. develop optimal solutions for the first stage of the converging-branch systems; and

6. utilizing the output of the first stage as input to the second decision point, repeat the routing process as presented in Step 4 and optimization process in Step 5 until all stages have been calculated.

An example of the sequencing of the tableau formations for the upper portion of the Potomac River Basin, shown in Figure 5, is given in Table 1. The output of the computer program is a series of incremental flow and dynamic programming tableaux. The incremental flow tableaux contain input information including all dependent states for the entire flow ranges of the regulated systems required for the optimization process. The dynamic programming tableaux consist of the results of the process for the given decision point plus the dependent states for all feasible solutions.

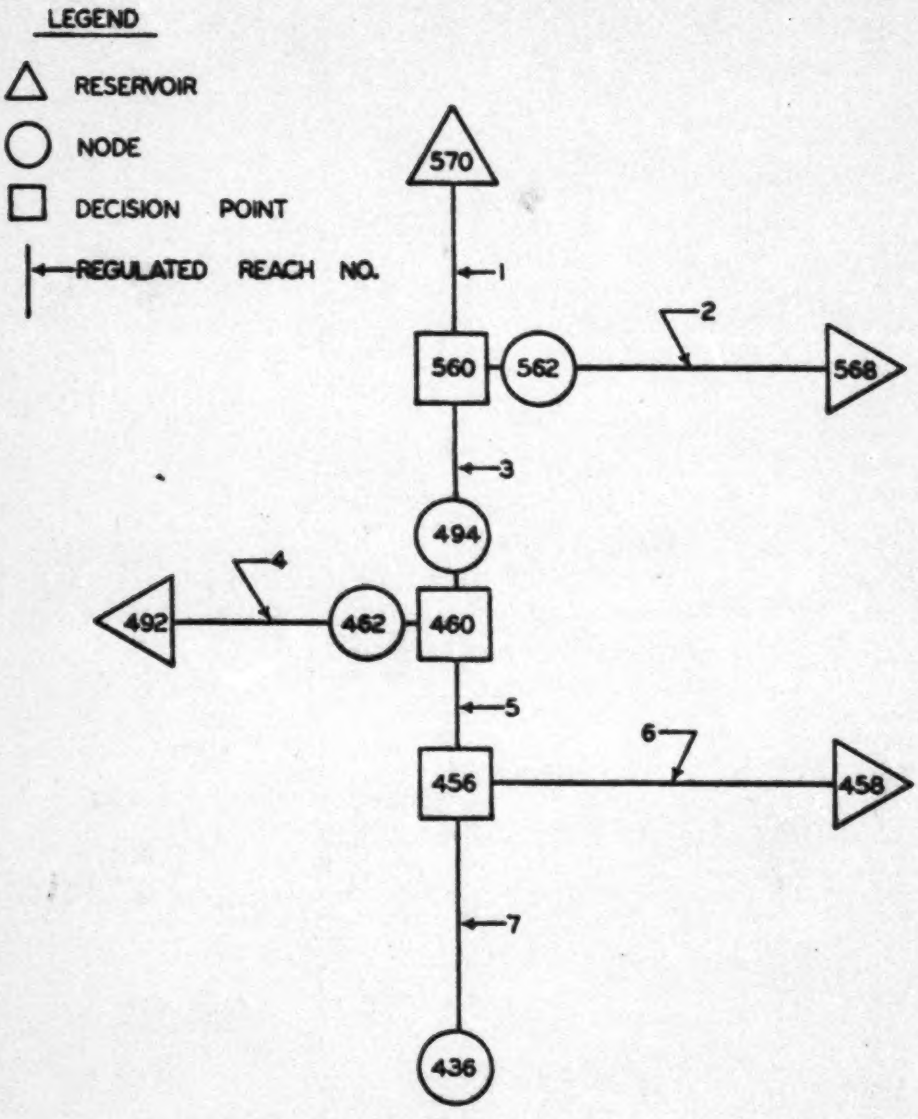


Figure 5 - Staging Process and Tableau Formulation for the Upper Portion of the Potomac River Basin

Table 1

TABLEAU FORMATION UPPER POTOMAC RIVER BASIN

Stage Input			Stage Output	
Incremental Flow Tableaux			Dynamic Programming Tableaux	
Index Node	Reach		Index Node	Tableaux No.
570	1	→	560	1
562	2			
494	3	→	460	2
462	4			
460	5	→	456	3
458	6			
436	7	→	To be continued downstream	
. . . . .				

THE POTOMAC SYSTEM

The field testing and analysis of the flow-release model was conducted on the non-tidal portion of the Potomac River Basin. In the non-tidal portion of the basin, which has a drainage area of over 11,000 square miles, there are 16 proposed and two existing reservoirs.

Figure 6 is a general map of the Potomac River Basin, including the proposed reservoir system. Data are presented in Table 2 for 14 of the 16 proposed impoundments for the Potomac system. The Mount Storm and Savage II impoundments, shown in Figure 6, are in series with the Bloomington and Savage I reservoirs, respectively, and therefore were not included in the analyses. If the proposed impoundments were in operation, the 7-day low-flow with a recurrence interval of once-in-ten-years would be increased from 705 cfs to over 2500 cfs at Washington, D. C.

For the model evaluation phase of this study, stream reaches receiving significant wastewater discharges and/or those subject to flow regulation have been incorporated into the system. Included in the wastewater inventory are all discharges with a flow equal to or greater than 0.5 mgd, or a population equivalent to or greater than 1,000. A summary of the flow-release model inputs is presented in Table 3. The wastewater discharges included in the model represent over 90 percent of the total BOD load to the stream. All

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major water water supply intakes in the non-tidal portion of the Potomac Basin were also incorporated into the model.

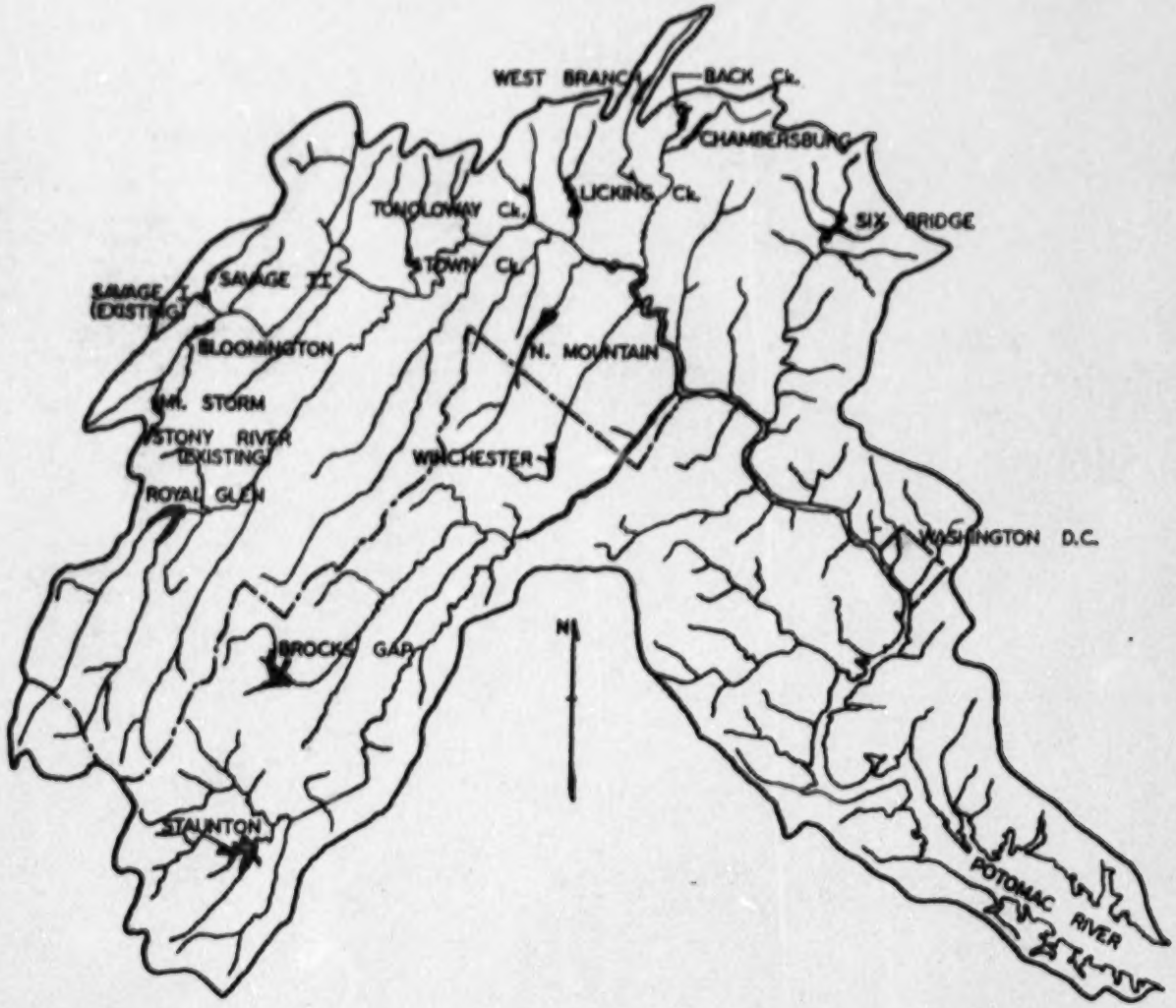

Figure 6 - A Map Showing the Planned Reservoir System for the Potomac River Basin

Table 2

RESERVOIR DATA

Potomac River Basin

Name	Node	Total Storage (Acre-Ft)	Storage for Water Quality Control (Acre-Ft)	Total Cost ($1000)	Water Quality Control Cost ($1000)	Average Stream Flow (cfs)	Increase In Dependable Flow (cfs)
Bloomington	570	137500	92000	50865	22703	476	212
Royal Glen	492	338000	228000	30953	15744	684	484
Chambersburg	402	42800	28000	12595	9468	163	63
Staunton	244	143000	104000	24363	16485	276	163
Six Bridge	56	103000	65000	14601	6487	350	104
W. Branch	393	77500	45000	11893	5717	90	65
Brocks Gap	156	187000	120000	16641	9477	185	147
Winchester	356	77000	28300	11132	5631	77	45
Licking Creek	428	120500	82100	11528	5065	166	138
Town Creek	458	96800	57500	10009	3066	150	97
N. Mountain	420	195000	95500	16655	4655	183	137
Savage	566	50000	39000	13233	4018	73	48
Back Creek	398	46900	19700	7587	3711	72	28
Tonoloway Creek	434	88000	50000	14086	4461	118	84

Table 3

A SUMMARY OF FLOW-RELEASE MODEL INPUTS FOR THE POTOMAC RIVER BASIN

Entity	Quantity
Wastewater Discharges:	
Organic	64
Thermal	8
Stream Flow Addition Points:	
Regulated	14
Unregulated	11
Increments of Drainage Area	56
Surface Water Supplies	26
Stream Segments	307
Stream Miles	894
Existing Impoundments	2
Proposed Impoundments	14

RESERVOIR RELEASE SEQUENCES

From the tableaux, it is possible to determine two types of optimal reservoir release sequences: (1) the "best" water quality for a given flow requirement, or (2) the minimized release rates for a given water quality requirement. These releases are formed from the system of sub-problems and solutions in a reverse order of the converging staging process. Patterns of optimal release for 1000, 1500, and 2000 cfs flow targets at the Potomac Estuary are presented in Figure 7.

These release patterns have been developed employing the following conditions:

1. 1966 wastewater loadings at 85% BOD_5 removal and 1966 water supply withdrawals;

2. the BOD and DO of all stream flow tributaries and addition points including reservoirs were assumed to be 2.0 and 8.0 mg/l, respectively;

3. O'Connor's K_2 formulation was used to determine reaeration rates;

4. steady-state temperature for the entire river system was 23°C;

5. base flow and regulation capacity of the reservoirs are as indicated in Figures 7 and 8; and

6. a DO constraint of 4.0 mg/l in all reaches.

The above have been used also as standard test conditions for the analysis of the optimization parameters in conjunction with 1500

cfs requirement at the estuary.

For the preceding conditions, the Savage II and Brocks Gap reservoirs at nodes 104 and 568 are at their maximum release rate for all three target flows at the estuary while the Tonoloway Creek and Six Bridge reservoirs at nodes 56 and 434 are at their base flow rates. The two reservoirs which are at their maximum rates indicate a primary importance in the optimal flow release patterns. The Staunton, Winchester, Chambersburg, Back Creek, and West Branch reservoirs at nodes 244, 356, 393, 398, and 402, respectively, are utilized to capacity at 1500 and 2000 cfs target, indicating a secondary importance in the release pattern.

In the entire Potomac River Basin affected by the proposed reservoir system, the North Branch was the only section that required additional flow regulation to meet the 4.0 mg/l DO objective. This was reflected in the constant release rates from Savage II and Bloomington reservoirs for all targets and levels at the estuary.

With the DO deficit and BOD optimization parameter (Figure 7) only one solution was normally retained per flow increment even when a BOD grid as small as 0.5 mg/l was employed. The retention of a single solution per flow increment essentially reduces the optimization to a single return variable.

More solutions were retained with a smaller BOD grid mainly in the upstream reaches of the Potomac. However, these additional feasible solutions were eliminated at the lower decision points.

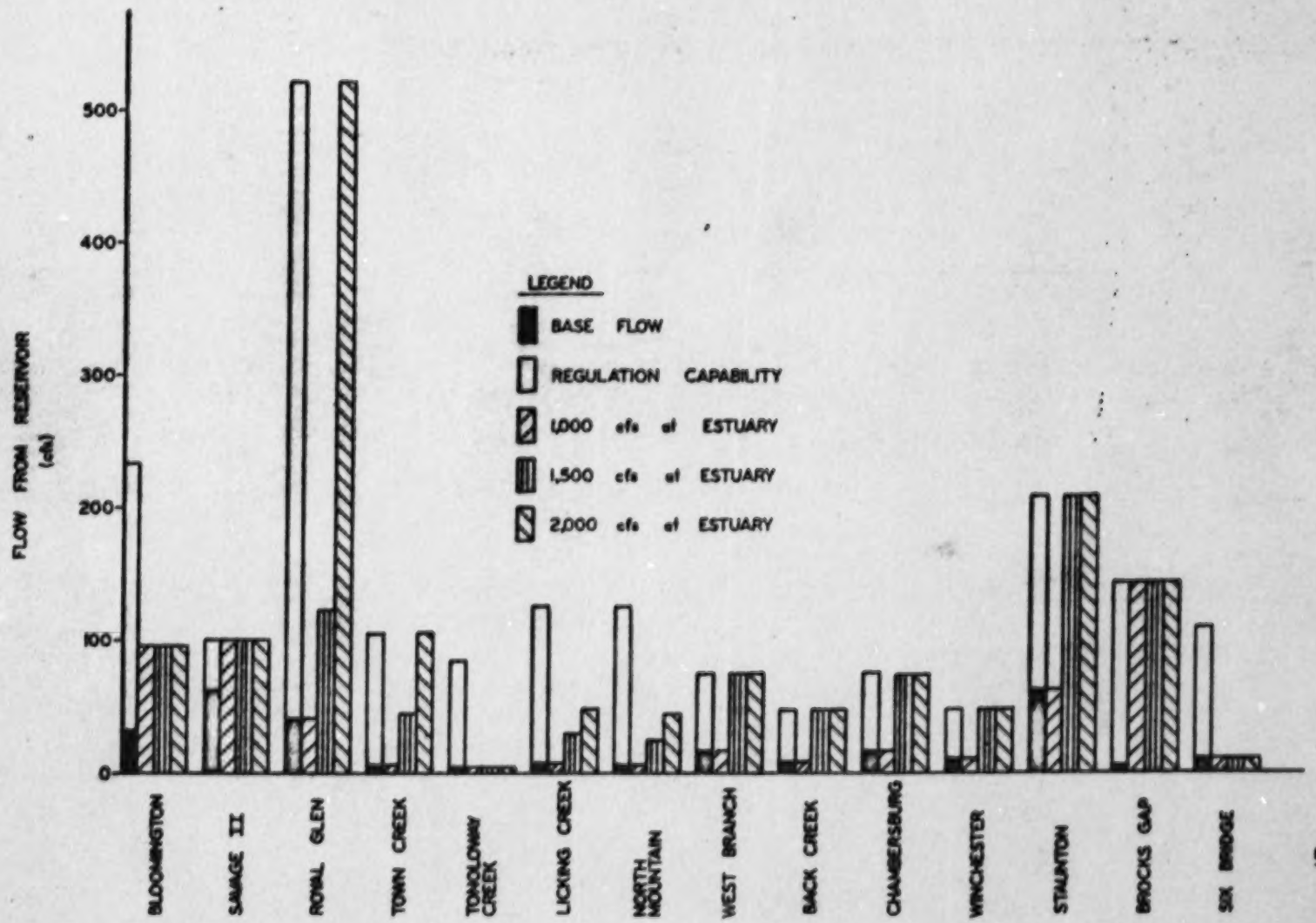

Figure 7 - Optimal Reservoir Release for Various Flows at the Potomac Estuary

To investigate the effect of the optimization parameter on the release sequence, cost of flow regulation was added to the flow-release model as the second return variable. The cost of flow regulation from each reservoir was read in as discrete units of dollar increase per unit increase in flow. Cost data were obtained from the uniform-use-rate versus reservoir cost curves as developed by the Corps of Engineers [16].

The release rates for 1500 cfs at the estuary for the optimization parameters of: (1) BOD and DO deficit, (2) BOD, and (3) BOD and least cost of reservoir storage are presented in Figure 8. While the release sequences are very dependent on the choice of optimization criteria, the effect on the water quality entering the estuary is minimal.

Concentrations of both BOD and DO for the flow range of 700 to 2000 cfs at the estuary varied only about 0.5 mg/l for the various feasible solutions. For many of the upper stream decision points, the small range of BOD and DO deficit concentration differentials for a given flow state in the optimization procedure has been one of the most significant observations in the application phase of the study.

This finding indicates that the solutions of water quality problems in most of the sub-basins are independent of one another. Thus the optimal reservoir flow release sequence based solely on maintaining the best water quality at the Potomac Estuary for a given flow requirement appears to have little significance in

formulating a reservoir release policy. It should be pointed out, however, that this conclusion is specific to the particular test basin. Different conclusions are likely for different types of river basin systems.

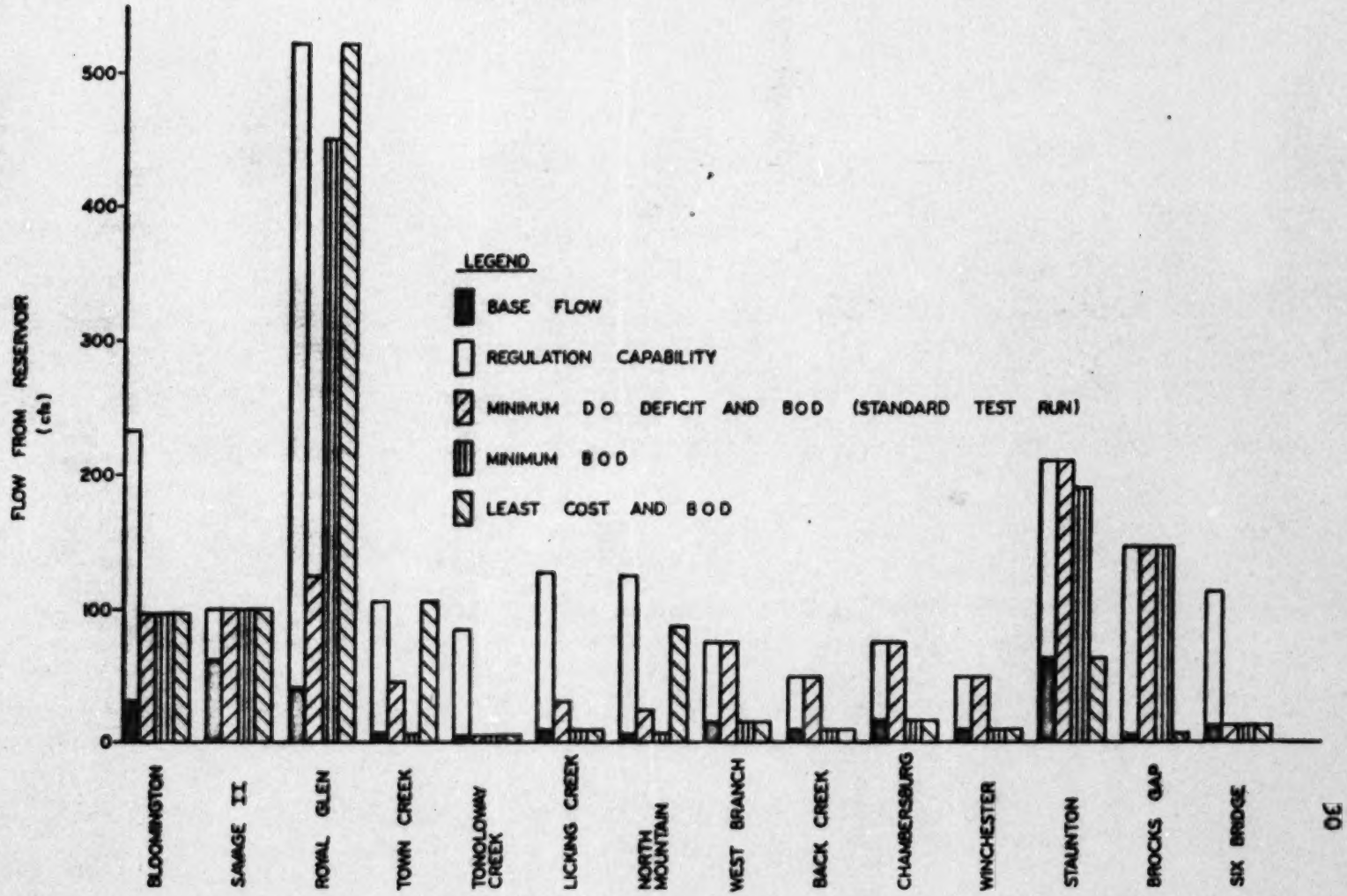

Figure 8 - Optimal Reservoir Release for Various Optimization Parameters for a Flow Requirement at 1500 cfs at the Estuary

SUMMARY AND CONCLUSION

A general mathematical model for determining optimal flow requirements from multiple reservoirs for water quality control has been developed. The optimal flow-releases are formed using the converging-branch multistage system of dynamic programming.

At each stage or decision point, optimal release sequences based on single and dual return variables. The model incorporates formulations for DO, BOD, temperature, reaeration, and physical relationships of stream velocity and depth.

Optimal reservoir release sequences from the proposed impoundments in the Potomac River Basin have been developed for three flow targets for the fresh water flow into the estuary. The release sequence was found to be very dependent on the choice of optimization criteria.

The small range of BOD and DO deficit concentration differentials for a given flow state in the optimization procedure was one of the most significant findings of the study. This observation demonstrates the need for well defined and verified water quality formulations. It also indicates that most of the water quality problems in the sub-basins of the Potomac are independent of one another.

From the output of the model, water resources planners can investigate the effect that various water quality formulation coefficients and optimization parameters may have on release rates. For example; in the Potomac River Basin, the choice of

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optimization criteria was shown to have considerable effect on the release for the given flow rate at the estuary.

The ability to control the number of feasible solutions, which are carried forward in a converging concept by proper BOD grid and flow increment selection, makes the enumeration process a powerful optimization procedure. The enumeration process can be used with noncumulative as well as cumulative return variables. The need for continued or smooth return functions is eliminated as required in most linear programming solutions.

At each stage, it is possible to relate the effect of the decision process to other downstream stages. These added functions of the model provide a mechanism for evaluating possible "trade-offs" within the system.

APPENDIX-NOTATION

The following symbols are used in this paper:

a, c = a constant

b, d = an exponent

BOD = biochemical oxygen demand

DO = dissolved oxygen

D(0) = dissolved oxygen deficit at the beginning of a reach

D(t) = dissolved oxygen deficit at the end of a reach

D_L = coefficient of diffusion of oxygen in water

H = depth of stream

K = reaction rate

K_t = reaction at temperature T

K_{20} = reaction rate at 20°C

MBOD = BOD state of M for a given MFLOW

MDOD = DOD state of M for a given MFLOW

MFLOW = flow state of M

MK = deaeration rate of M for a given MFLOW

MT = temperature state of M for a given MFLOW

NBOD = BOD state of N for a given NFLOW

NDOD = DOD state of N for a given NFLOW

NFLOW = flow state of N

NK = deaeration rate of N for a given NFLOW

NT = temperature state of N for a given NFLOW

OBOD = BOD state of O for a given OFLOW

APPENDIX-NOTATION (cont.)

ODOD = DOD state of O for a given OFLOW

OFLOW = flow state of O

OK = deaeration rate of O for a given OFLOW

OT = temperature state of O for a given OFLOW

P = $e^{-(K_2 t/2)}$, reaeration term

Q = stream flow

SAT DO = DO saturation concentration for a given temperature T

T = temperature

t = travel time

Ts_1 = steady-state temperature at the end of a reach

Ts_0 = steady-state temperature at the beginning of a reach

T_1 = predicted temperature at the end of a reach

T_0 = temperature at the beginning of a reach

V = stream velocity

W = temperature coefficient

X = distance

Y = $L_0 (1 - e^{-K_1 t})$, oxygen uptake term

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